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ARTICLE

Relationship between Tension and Frequency of a Violin String

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To produce sound, vibration is required; the pitch a violin produces comes from the vibration of its strings, and these vibrations require near-constant frequencies to maintain a constant pitch. Frequency is the measure of how many oscillations happen per second, and it ultimately determines the pitch. In acoustic instruments, such as a violin, the stable vibrations are produced by waves (Music Acoustics, 1997), (Kognity, 4.5.2). A violin has 4 strings: the G, D, A, and E strings, with their pitches and thickness increasing from low to high in that order. Each string is firmly anchored to a violin at two farthest ends: the peg and the tailpiece. The peg can be rotated to change the tension of the string to increase or decrease the pitch of the string. The tailpiece is set stationary to allow only one end (the peg) to alter the tension of the string. The waves produced in the strings are fixed-end waves, meaning there are two nodes at each end. In a violin, the nut and the bridge act as the fundamental nodes for the vibration of the strings. The nut and the bridge sets the vibratable length of the string, meaning that only the length between the nut and the bridge vibrates to produce the pitch (Kognity, 4.5.2).

INTRODUCTION

To make the string vibrate in order to produce a pitch, an external source of energy is required. In a violin, one source of external energy is the finger. By plucking the string, the technique known as pizzicato, the string is removed from its equilibrium position, so the energy exerted from the string returning to the equilibrium position sends a pulse down the string. The pulse travels down the string to the node (nut or bridge) and reflects its orientation after contacting the node; the pulse returns down the string to the other node, and similarly to the prior, reflects its orientation again and sends the pulse back. This repetition of the pulse travels from node to node across the string creates the wave with the fundamental frequency. The pulse displaces the air around the string, producing a longitudinal wave. Since the string is vibrating at a constant frequency, the air around is displaced in the same frequency, producing sound. In a violin string, this cycle repeats numerous times until the frequency is great enough that it becomes an audible pitch (Kognity, 4.5.1; Kognity, 4.2.3; Music Acoustics, 1997).

The pitch of the note, or ultimately the frequency, depends on four characteristics: the length of the vibrating string, mass of the vibrating string, tension, and harmonics of the string. The vibrating string is the part of the string that vibrates, meaning the part in between the two fundamental nodes (nut and bridge) at each end (Music Acoustics, 1997).

With all of these characteristics in mind, the formula for the frequency is:

$$\textbf{(Eq. 1)}\ f_n = \frac{n}{2}\sqrt{\frac{F}{LM}}$$

where 'n' is the fundamental frequency, 'F' is the tension force, 'M' is the mass of the vibrating string, and 'L' is the length of the vibrating string. Since the violin strings vibrate at its fundamental frequency when played as an open string (meaning no other contact is on the string, such as a finger placement), the value of 'n' will be 1 for the equation (Music Acoustics, 1997).
Hence, the formula for violin string's frequency is:

$$\textbf{(Eq. 2)}\ f = \frac{1}{2}\sqrt{\frac{F}{LM}}$$

The purpose of this experiment is to explore the relationship between the tension applied to a violin string and the resulting frequency of a violin string in reference to **(Eq. 2.)**

RESEARCH QUESTION

How does changing the tension (N) affect the frequency (Hz) of a violin string?

MATERIALS

- Violin String - 'D' String x1
- LoggerPro
- Wheeled Pulley x 1
- Clamp Stand x 1
- Masses of (0.5kg, 1kg, 1.5kg, 2kg,
- 2.5kg, 3kg)
- Digital Instrument Tuner
- Markers
- Measuring Tape
- Clamps x 2

PROCEDURE

The tailpiece end of the string was fastened to the clam-stand, and then a mark was placed 32.5 cm along the string starting from the clamp-end (balled end) of the violin string to identify the vibratable string length. On the other end of the string, a mass of 0.5kg was tightly tied to start. Then, a pulley was set up on the edge of a flat surface, and using two clamps, the pulley was stabilized. The mass-end of the string was placed overthe pulley such that the mass hung over the edge. To straighten the string, the clamp-stand was positioned away from the pulley such that the 32.5 cm mark on the string was on the peak of the pulley. As such, the pulley and the clamp will act as nodes for the experiment. To stabilize the position of the stand, additional masses were placed on the stand so that no further unintentional slides or movements could happen to disturb the 32.5 cm vibratable string length. Then, the height of the clamp was adjusted such that the peak of the pulley and the clamp-end string were at an equal height. A digital tuner on a phone was turned on and prepared, with the A_4 frequency set to 440 Hz, which is the common reference pitch of many classical musicians. It was then placed directly below the center of the string. Then, the string was plucked directly downwards from 1 cm above the string, and the digital tuner measured the frequency produced by the vibrating string. After finishing 6 trials of 0.5 kg mass, the mass was swapped with 1.0 kg, 1.5kg, 2.0kg, 2.5kg, and 3.0kg masses. For each mass, the string was plucked 6 times. At the end of the experiment, there were a total of 36 trials.

RESULTS

Graph 1 is a Frequency against Tension Force graph with data points of frequencies
produced by the string with varying tension forces acting on the string from **Table 2**. Recall formula for frequency,

$$\textbf{(Eq. 2)}: f = \frac{1}{2}\sqrt{\frac{F}{LM}}$$

Since L, the vibratable length, and M, the mass of the vibrating string, are controlled variables, these two components of the formula are constants. Therefore, the formula can
be altered:

$$f = \frac{1}{2}\sqrt{\frac{F}{LM}} = \frac{1}{2\sqrt{LM}}\sqrt{F}.$$

$$\textbf{(Eq. 3)}: f = \frac{1}{2\sqrt{LM}}\sqrt{F}$$

In this new formula, $\frac{1}{2\sqrt{LM}}$ would be a constant, and F would be the independent variable. Since the frequency formula is an exponential function when letting F as the independent variable, I chose to curve-fit the model with an exponential equation
(**Eq. 4**) $y = A * X^B$, which follows the structure of the formula, since A, a constant, would theoretically equal $\frac{1}{2\sqrt{LM}}$, and B, also a constant, would would theoretically equal 0.5. The LoggerPro program calculated the constants A and B and calculated the equation of:

$$\textbf{(Eq. 5)}: f = 46.13 * x^{0.4971}$$

DISCUSSION

Through the experiment, it has been concluded that changing the tension on a violin string affects the frequency produced by the string through an exponential relationship. Graph 1 of the processed data resulted in the equation of (**Eq. 5**) $f = 46.13 * X^{0.4971}$ [based on the model equation (**Eq. 4**) $y = A * X^B$] where 'f' is the frequency produced by the violin string, and 'x' is the tension force. This equation has a positive correlation
coefficient of 0.9995, which shows that the data points have a very strong positive relationship. The curve generated by the curve-fit was all inside the region of the error bars of each data point, which shows that the graph is relatively accurate. The very strong positive correlation of the curve-fit shows that the processed data points are strongly related and substantiates the conclusion that the tension on a violin string positively affects the frequency produced by the string.

The resulting equation closely matched the theoretical formula of the frequency of the string, (**Eq. 3**) $f = \frac{1}{2\sqrt{LM}}\sqrt{F}$. In this formula, L is the vibratable length of the string, which was controlled to be 0.325 m. Similarly, M is the mass of the vibrating string was $3.58 * 10^{-4}$ kg, which was dependent on the length of the vibrating string, a controlled variable. Since the vibrating string was set constant, the mass of the vibrating string was
also held constant, derived from the given linear density and vibrating string length (*Calculation 4*). Since L and M are both constants, the term $\frac{1}{2\sqrt{LM}}$ becomes a constant that can be equated to:

$$\frac{1}{2\sqrt{0.325*3.58*10^{-4}}} = 46.35$$

by substituting the respective values. This constant, 46.35, is equivalent to the constant A in the model equation, (**Eq. 4**) $y = A * X^B$. The equation (**Eq. 5**) derived from the curve-fit of the processed data points resulted in the A constant to be 46.13 with an
absolute error of 1.28, giving the value of A a minimum and maximum of 44.85 & 47.41. Since 46.35 is between 44.85 and 47.41, it can be concluded that these two constants are
equivalent due to the uncertainty. The equivalence of the A constant substantiates the relationship between the tension and the frequency.

The other factor, $\sqrt{F}$, of the formula (**Eq. 3**) $f = \frac{1}{2\sqrt{LM}}\sqrt{F}$

is equal to $F^{0.5}$. This constant, 0.5, is equivalent to the constant B in the model equation (**Eq. 4**) $y = A * X^B$. The equation (**Eq. 5**) derived from the curve-fit of the processed data points resulted in the constant B to be 0.4971 with absolute error of 0.0092, giving the constant the minimum and maximum of 0.4879 & 0.5063. Since the constant 0.5 from the theoretical formula is within 0.4879 and 0.5063, it can be concluded that these two constants are equivalent due to the uncertainty. The equivalence of the B constant substantiates the relationship between the tension and the frequency. Because the theoretical formula, **(Eq.3)** $f = \frac{1}{2\sqrt{LM}}\sqrt{F}$, and the derived equation of the data points, **(Eq. 5)** $f = 46.13 * x^{0.4971}$, closely match, shown by the equivalence of both 0.4971 constants A and B, and the correlation coefficient was 0.9995, which shows that the data points have a very strong positive relationship, it can be concluded that changing the tension on a violin string affects the frequency produced by the string, and there is a very strong positive relationship between these two variables.

WEAKNESSES AND IMPROVEMENTS

One of the errors in the experiment was the variation of the length of the vibrating string. This length was set constant to 32.5 cm, and an effort to keep this length constant was done by measuring and drawing a mark at 32.5 cm from one end of the string. Before measuring the frequency, an effort was made to place the 32.5 cm mark directly on top of the peak of the pulley to act as the node. However, the string is not perfectly straight: there are noticeable and unnoticable curves, bends, and coils on the string macroscopically and microscopically that makes it difficult to measure the exact length of the string. This means that the measured 32.5 cm is not accurately 32.5 cm due to the variances of the string itself due to bends and curves. This would have caused an error in the frequency measured because the length of the vibrating string would have been different.

In addition, when the string was tied to a greater mass, the increasing tension would have allowed the string to straighten progressively and in greater magnitudes than the previous trials, causing the string to elongate. Since the mark on the string - 32.5 cm - was originally marked when the string was not affected by such tensions and stretches, the string won't be 32.5 cm at the mark anymore. Instead, the mark would represent a length greater than 32.5 cm because the string itself would have elongated while the mark would remain in the same place on the string, so the mark's position wouldn't be accurate.It is also important to note that the mass of the vibrating string is dependent on the vibrating string. Since the string I used has a linear density set to 1. 10 * 10^{-3} kg/m, change in vibrating length will cause a change in the mass. Because the vibrating string length is now longer, the mass of this length will increase, meaning that the length and mass would have increased. As seen in the formula, **(Eq. 2)** $f = \frac{1}{2}\sqrt{\frac{F}{LM}}$, length and mass, the measured frequency would be relatively lower, causing an accuracy error. These errors can be minimized by remeasuring and remarking the string before every trial while the force of tension is acting on the string. The tension force will allow the string to stretch, and the string will be stretched throughout the trial, meaning it's magnitude of elongation will most likely be constant. Therefore, if I remeasure the 32.5 cm of vibrating string length while the string is already stretched due to the corresponding mass in each trial instead of using the original 32.5 cm mark from the beginning of the experiment, the string length will be relative to the instant tension and will become more accurate.

Another error that affected the measurements was the slight variations in the digital tuner affecting taking in the sound the string produced to measure the frequency. The digital tuner was very sensitive that any slightest sound interfered against the sound of the string. After the string had been plucked, the reading on the digital tuner would sometimes change a few times, which meant that some data might not be as accurate. The interference could have occurred by the nature of the room and walls the experiment was held in, as the sound could have reverberated against the wall. This error would have caused some data to be greater and some to be less than the actual frequency of the string at different tensions. A viable solution to this error would be to perform the experiment in a soundproof room to prevent echoing and reverberation of extra sound that could interfere with the accuracy of the digital tuner.

STRENGTHS AND EXTENSIONS/FURTHER STUDY

I believe that the independent variable range choices were well-suited to this experiment. The values of the forces of tension represented the similar magnitudes of the forces the pegs acted on a violin spring, and the ranges allowed the violin string to vibrate at an audible pitch. These forces allowed the recreation of a simulation of an actual violin, which increased the audentity of the focus of the experiment. The range of forces also increased by appropriate intervals (increments of 0.5kg), which helped in visually noticing the non-linear relationship from the graph.

Another strength of the experiment that worked well was the materials and the collection of the data. During the experiment, the procedure and the set-up allowed a relatively straightforward collection of data without encountering many struggles. The clamps held the string and the pulley very firmly, which allowed little errors from the pulley and the stand. The secured string from the pulley and clamps allowed me to pluck the string without the equipment shifting or moving, which allowed the data that was collected to be relatively precise. In addition, taking 6 trials helped improve the precision of the frequencies, which also helped reduce the mean uncertainties. By taking more trials, I was able to reduce the uncertainty error, thus making the data more reliable. For future study, I would like to continue working with physics involving music and the violin. Since this experiment studied the fundamental frequency of the violin strings based on tension, one pathway for future study is to compare frequencies of different strings with different linear densities and material, such as the G, D, A, and E strings of a violin, or even comparing strings from different instruments, following the similar structure of this experiment. Since thicker strings such as the violin G string can support more tension than the string I used in this experiment, this would also allow me to expand my range of the independent variable, which would allow me to cover more diverse pitch ranges.. Another pathway is to investigate the effect of the pitch produced by drawing the bow instead of plucking the string at different locations on the string. In relation to the bow, it could also be interesting to investigate how the speed and tension of the bow drawn can affect the amplitude/loudness of the string.

ACKNOWLEDGEMENTS

Because my mother is a violinist, I grew up immersed in the world of music. Watching her as I grew older, I always dreamed of playing the same instrument as her, which eventually was granted when I was 6 years old. For almost the entirety of my years of playing, I simply accepted the fact that turning the pegs of a violin sharpened or flattened the pitch of the strings. However, as I started to learn more about the properties of waves and frequencies, I gained a deeper insight and connection between music and physics, which sparked my inquiry on how the strings of the violin produce notes and pitches.

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APPENDIX

Image 1: *Labels of the Violin Parts*

Raw Data

Table 1. *Experimental Data of Frequency (Hz) of the Violin String due to Changes in Tension Force (N)*

	Frequency (Hz)					
Force (N)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6
4.9	102.1	103.6	102.4	100.9	102.5	102.1
9.8	142.8	145.6	144.0	143.9	146.1	143.6
14.7	173.1	175.5	174.8	176.3	177.3	174.9
19.6	201.2	201.2	199.2	201.0	198.3	202.9
24.5	221.2	223.3	227.2	224.8	225.1	224.1
29.4	251.1	253.2	250.7	246.9	250.6	249.1

Qualitative Data:

- As heavier mass was tied to the string, the overall perceived pitch rose/sharpened.
- The perceived notes, by ear, were approximately G#2, D3, F3, G#3, A3, B3, respectively in the order of increasing tension force.
- The perceived notes didn't sharpen by the same musical note intervals (half steps, whole steps) from increased mass tied
- At heavier masses, the notes went up by only a half step or whole steps, while at earlier masses, the intervals were more distinct and largely noticeable.
- A formation of a standing wave could be noticed with the pulley and clamp acting as the two nodes.

Data Processing

Table 2. *Mean Frequency (Hz) of the Violin String from changing Tension Force (N)*

Force (N)	**Mean Frequency (Hz)**
4.9	102.3 ± 1.4
9.8	144.3 ± 1.7
14.7	175.3 ± 2.1
19.6	200.6 ± 2.3
24.5	224.3 ± 3.0
29.5	250.3 ± 3.2

Graph 1. *Frequency (Hz) Produced by the Violin String from Increasing Tension Force (N)*

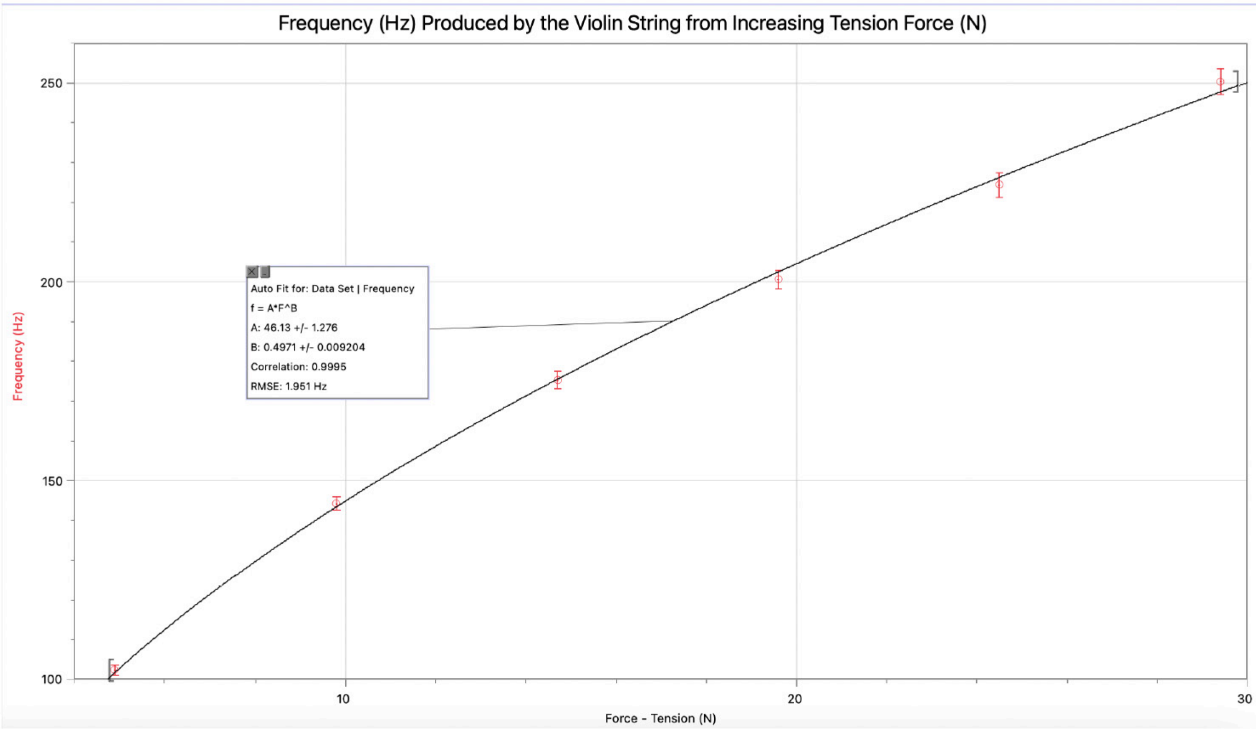
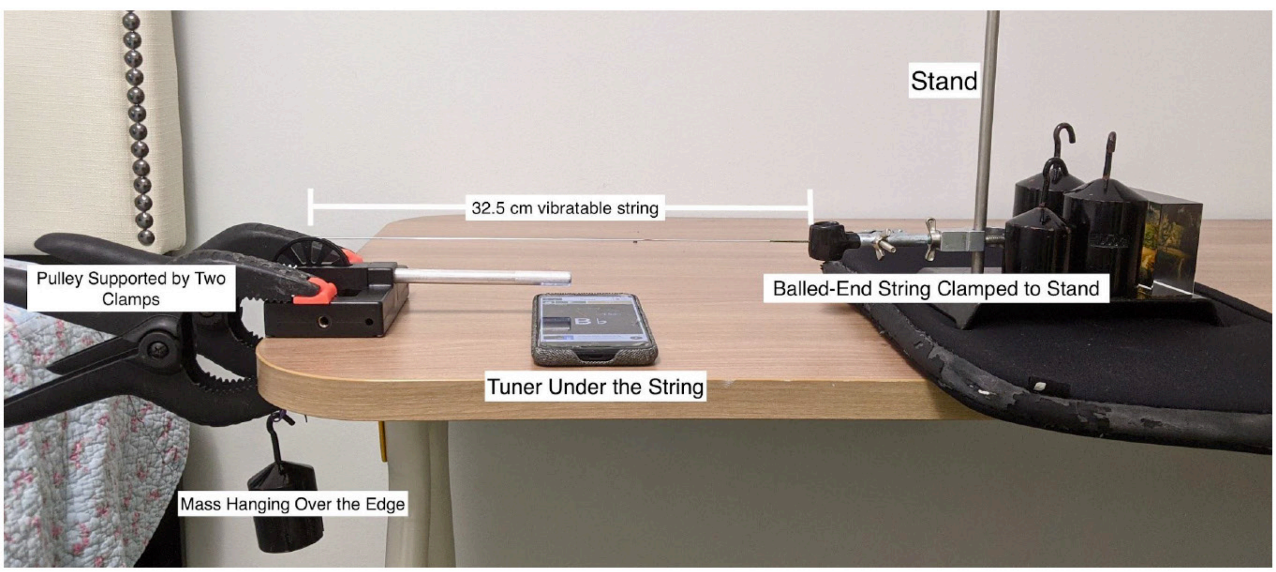


Image 2: *Setup of the Experiment*



(Photo of the Setup of the Experiment)

JONGMIN (BRIAN) LIM

My name is Jongmin (Brian) Lim, and I am in 12th grade at Glenlyon Norfolk School this 2021-2022 school year. One special aspect of my life is that I've lived in South Korea, Canada, and the US, which were all very unique experiences that helped shape who I am today. Throughout my journey of learning in these countries, I developed a strong passion for the sciences, including all physics, biology, and chemistry. With my strong background in mathematics, these intellectual interests synergized nicely, and I've used such intellectual motivations to pursue more explorations of my curiosity in the sciences.